Teacher notes
Topic A

A classic energy and momentum problem

A cart X of mass 2.0 kg moving at 5.0 m s^{-1} approaches a stationary cart Y of mass 3.0 kg. A spring of spring constant 1.2×10^4 N m^{-1} is attached to the front of X.

After the collision X bounces back with speed 1.0 m s^{-1}.

(a) Calculate the speed of Y after the collision.
(b) Show that the collision was elastic.
(c) What is the maximum compression of the spring?

Parts (a) and (b) are straightforward. Conservation of momentum says

$$2.0\times5.0=-2.0\times1.0+3.0\times v_{\text{Y}}$$

and so $v_{\text{Y}}=4.0\,\text{m}\,\text{s}^{-1}$.

The initial kinetic energy is $K_{\text{i}}=\frac{1}{2}\times2.0\times5.0^2=25\,\text{J}$.

The final kinetic energy is $K_{\text{f}}=\frac{1}{2}\times2.0\times1.0^2+\frac{1}{2}\times3.0\times4.0^2=25\,\text{J}$.

$K_{\text{i}}=K_{\text{f}}$ so the collision was elastic.

Part (c) is more interesting.

X will change its velocity from 5.0 m s^{-1} to – 1.0 m s^{-1} and Y will change its velocity from 0 to 4.0 m s^{-1}. During this time, the spring is compressed and so carries elastic potential energy. So kinetic energy

during this time is not conserved: some gets transferred to elastic potential energy. At maximum compression, the elastic potential energy is a maximum and so the amount of kinetic energy transferred is also a maximum. When do we have maximum kinetic energy transfer during a collision? The answer is when the bodies stick together and move as one. Momentum conservation always applies and so, at maximum compression,

$$2.0\times 5.0=(2.0+3.0)\times v$$

and so $v=2.0\ \text{m s}^{-1}$.

The change in kinetic energy is

$$\Delta K=25-\frac{1}{2}\times 5.0\times 2.0^2=15\ \text{J}$$

and this is the elastic potential energy at maximum compression.

Then

$$15=\frac{1}{2}\times 1.2\times 10^4\times x^2\ \Rightarrow\ x=0.05\ \text{m}.$$

(d) What is the impulse delivered to (i) X, (ii) Y and (iii) the system of X and Y?

(e) The collision lasted for 31 ms. What is the average force exerted on (i) X and (ii) Y?

(d) (i) $J_X=\Delta p_X=-2.0\times 1.0-2.0\times 5.0=-12\ \text{N s}$

(ii) $J_Y=\Delta p_Y=3.0\times 4.0-0=12\ \text{N s}$

(iii) $J_{\text{total}}=\Delta p_X+\Delta p_Y=-12+12=0$

(e) (i) $F_X=\frac{\Delta P_X}{\Delta t}=-\frac{12}{31\times 10^{-3}}=-387\approx -390\ \text{N}$, (ii) $F_Y=\frac{\Delta P_Y}{\Delta t}=+\frac{12}{31\times 10^{-3}}=387\approx 390\ \text{N}.$