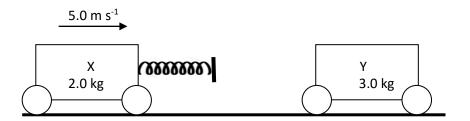
Teacher notes Topic A

A classic energy and momentum problem

A cart X of mass 2.0 kg moving at 5.0 m s⁻¹ approaches a stationary cart Y of mass 3.0 kg. A spring of spring constant 1.2×10^4 N m⁻¹ is attached to the front of X.



After the collision X bounces back with speed 1.0 m s⁻¹.

- (a) Calculate the speed of Y after the collision.
- (b) Show that the collision was elastic.
- (c) What is the maximum compression of the spring?

Parts (a) and (b) are straightforward. Conservation of momentum says

$$2.0 \times 5.0 = -2.0 \times 1.0 + 3.0 \times v_{y}$$

and so $v_{y} = 4.0 \, \text{m s}^{-1}$.

The initial kinetic energy is $K_i = \frac{1}{2} \times 2.0 \times 5.0^2 = 25 \text{ J}.$

The final kinetic energy is $K_f = \frac{1}{2} \times 2.0 \times 1.0^2 + \frac{1}{2} \times 3.0 \times 4.0^2 = 25 \text{ J}.$

 $K_i = K_f$ so the collision was elastic.

Part (c) is more interesting.

X will change its velocity from 5.0 m s^{-1} to -1.0 m s^{-1} and Y will change its velocity from 0 to 4.0 m s^{-1} . During this time, the spring is compressed and so carries elastic potential energy. So kinetic energy

IB Physics: K.A. Tsokos

during this time is not conserved: some gets transferred to elastic potential energy. At maximum compression, the elastic potential energy is a maximum and so the amount of kinetic energy transferred is also a maximum. When do we have maximum kinetic energy transfer during a collision? The answer is when the bodies stick together and move as one. Momentum conservation always applies and so, at maximum compression,

$$2.0\times5.0=(2.0+3.0)\times v$$

and so $v = 2.0 \, \text{m s}^{-1}$.

The change in kinetic energy is

$$\Delta K = 25 - \frac{1}{2} \times 5.0 \times 2.0^2 = 15 \text{ J}$$

and this is the elastic potential energy at maximum compression.

Then

$$15 = \frac{1}{2} \times 1.2 \times 10^4 \times x^2 \implies x = 0.05 \,\mathrm{m}.$$

- (d) What is the impulse delivered to (i) X, (ii) Y and (iii) the system of X and Y?
- (e) The collision lasted for 31 ms. What is the average force exerted on (i) X and (ii) Y?

(d) (i)
$$J_x = \Delta p_x = -2.0 \times 1.0 - 2.0 \times 5.0 = -12 \text{ N s}$$

(ii)
$$J_{y} = \Delta p_{y} = 3.0 \times 4.0 - 0 = 12 \text{ N s}$$

(iii)
$$J_{\text{total}} = \Delta p_{x} + \Delta p_{y} = -12 + 12 = 0$$

(e) (i)
$$F_X = \frac{\Delta P_X}{\Delta t} = -\frac{12}{31 \times 10^{-3}} = -387 \approx -390 \text{ N}$$
, (ii) $F_Y = \frac{\Delta P_Y}{\Delta t} = +\frac{12}{31 \times 10^{-3}} = 387 \approx 390 \text{ N}$.